PROVING TRIG IDENTITIES

Using: $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \operatorname{cosec}^2 x$,

and $\cos^2 x + \sin^2 x = 1$.

Examples: Prove that

1) $\frac{2\tan x}{1+\tan^2 x} = 2\sin x \cos x$

LHS = $\frac{2\tan x}{1+\tan^2 x} = \frac{2\tan x}{\sec^2 x} = 2\tan x \cdot \frac{1}{\sec^2 x} = 2\tan x \cdot \cos^2 x = 2 \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x$

= $2\sin x \cos x$ = RHS.

2) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$

LHS =$\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = \frac{(1-\sin\theta)+(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)} = \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta} = 2\sec^2 x$ = RHS.

3) $\operatorname{cosec}^2 x - 1 = \cos^2 x \operatorname{cosec}^2 x$

LHS = $\operatorname{cosec}^2 x - 1 = (1 + \cot^2 x) - 1 = \cot^2 x = \cot x \cdot \cot x = \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x}$

= $\cos^2 x \cdot \frac{1}{\sin^2 x} = \cos^2 x \operatorname{cosec}^2 x$ = RHS.

4) $\frac{\cot^2\theta - 1}{\cot^2\theta + 1} = 1 - 2\sin^2\theta$

LHS = $\frac{\cot^2\theta-1}{\cot^2\theta+1} = \frac{(\operatorname{cosec}^2 x-1)-1}{(\operatorname{cosec}^2 x-1)+1} = \frac{\operatorname{cosec}^2 x-2}{\operatorname{cosec}^2 x} = \frac{\operatorname{cosec}^2 x}{\operatorname{cosec}^2 x} - \frac{2}{\operatorname{cosec}^2 x} = 1 - 2\sin^2 x$

= RHS.

DELTA: Ex 34.1 pg 320